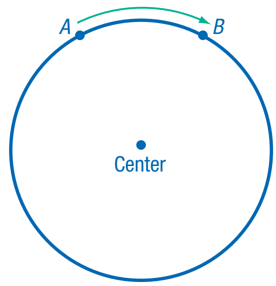
LESSON 4

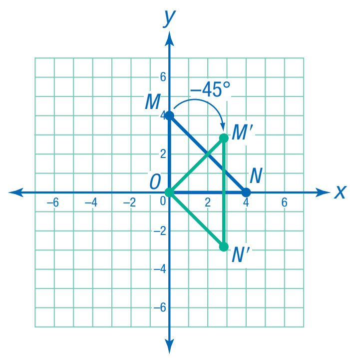
Rotations

UNDERSTAND A **circle** is the set of all points that are the same distance from a point called the center. Visualize turning the circle shown on the right so that point *A* moves onto point *B*. If you did that, the points would remain the same distance from the center, but they would each be in a different location.



A **rotation** is a transformation that turns a figure around a point, called the **center of rotation**. Just as with points on a circle, when you rotate a point around a center of rotation, it remains the same distance from the center of rotation. You can rotate a figure any number of degrees.

Counterclockwise is considered the positive direction, so the rotation shown on the right would be described as $-45°$ rotation around the origin. The same image could be obtained, however, by rotating the figure 315° clockwise, since $360 - 45 = 315$. So, this rotation could also be called a 315° rotation around the origin.



You can represent a rotation as a function for which the input is a coordinate pair. The output of that function is the image produced by the rotation.

A 90° rotation is equivalent to a $-270°$ rotation and has this function:

$$R_{90°}(x, y) = (-y, x)$$

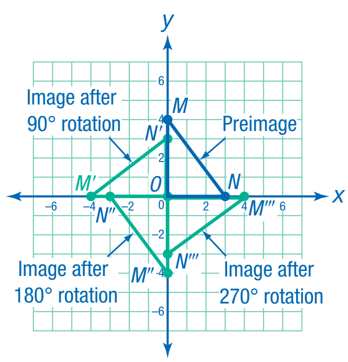
A 180° rotation is equivalent to a $-180°$ rotation and has this function:

$$R_{180°}(x, y) = (-x, -y)$$

A 270° rotation is equivalent to a $-90°$ rotation and has this function:

$$R_{270°}(x, y) = (y, -x)$$

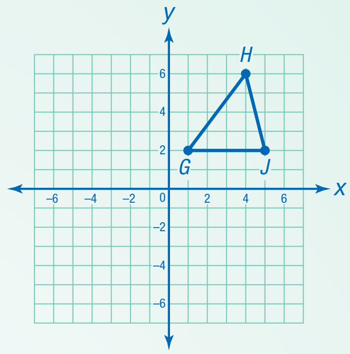
Compare the preimage on the right and its image after a 90° rotation or a 270° rotation. Notice that the hypotenuse of each of these images is perpendicular to the hypotenuse of the preimage. Corresponding sides of a figure and its image after a 90° or 270° rotation lie on perpendicular lines.



Now compare the preimage and its image after a 180° rotation. Notice that the hypotenuse of the image is parallel to the hypotenuse of the preimage. Corresponding sides of a figure and its image after a 180° rotation always lie on parallel lines or on the same line.

Connect

Triangle *GHJ* is graphed on the coordinate plane. Draw the image of this triangle after counterclockwise rotations of 90°, 180°, and 270° about the origin.



1 Apply the functions for the given counterclockwise rotations to the vertices of the triangle.

The vertices of $\triangle GHJ$ are $G(1, 2)$, $H(4, 6)$, and $J(5, 2)$.

The function that represents a 90° rotation around the origin is $R_{90°}(x, y) = (-y, x)$.

$R_{90°}(1, 2) = (-2, 1)$

$R_{90°}(4, 6) = (-6, 4)$

$R_{90°}(5, 2) = (-2, 5)$

The function that represents a 180° rotation around the origin is $R_{180°}(x, y) = (-x, -y)$.

$R_{180°}(1, 2) = (-1, -2)$

$R_{180°}(4, 6) = (-4, -6)$

$R_{180°}(5, 2) = (-5, -2)$

The function that represents a 270° rotation around the origin is $R_{270°}(x, y) = (y, -x)$.

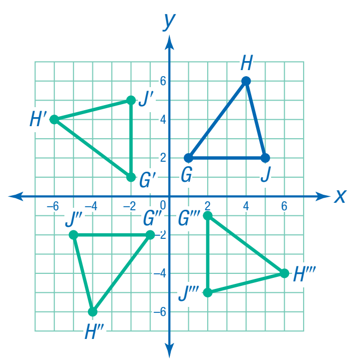
$R_{270°}(1, 2) = (2, -1)$

$R_{270°}(4, 6) = (6, -4)$

$R_{270°}(5, 2) = (2, -5)$

2 Graph and label each image.

Plot the vertices of each image, label them, and connect them.



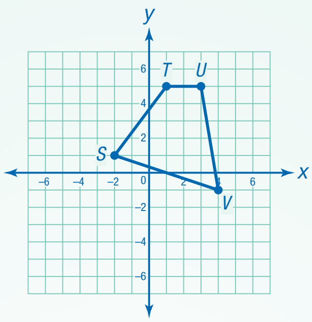
TRY

Identify the coordinates of the vertices of the images if the rotations had been in a clockwise direction.

EXAMPLE A Quadrilateral *STUV* is graphed on the coordinate plane. Transform quadrilateral *STUV* using this function:

$$R_\theta(x, y) = (-y, x)$$

Identify the degree measure of the rotation that the function performs. (Note: The Greek letter theta (θ) is often used to represent unknown angle measures.)



1 Identify the coordinates of the vertices of quadrilateral *STUV.*

The figure has vertices $S(-2, 1)$, $T(1, 5)$, $U(3, 5)$, and $V(4, -1)$.

2 Identify the coordinates of the rotated image and the degree measure of the rotation.

$R_\theta(x, y) = (-y, x)$, so:

$S(-2, 1) \rightarrow S'(-1, -2)$

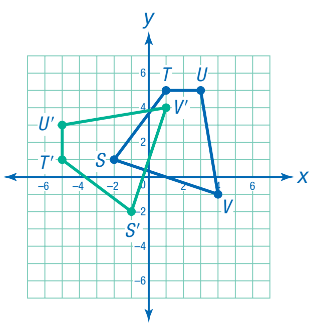
$T(1, 5) \rightarrow T'(-5, 1)$

$U(3, 5) \rightarrow U'(-5, 3)$

$V(4, -1) \rightarrow V'(1, 4)$

When the opposite value of *y* is taken and the values of *x* and $-y$ are switched, this indicates a 90° rotation.

3 Graph the rotated image and identify the transformation. Check your answer visually.



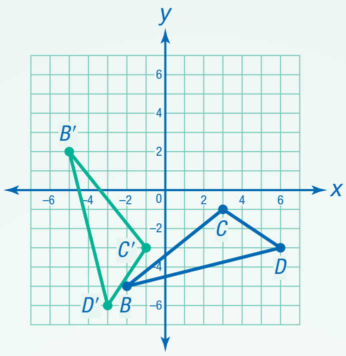
Eyeballing the graph confirms that this is a 90° rotation.

▶ $S'T'U'V'$ is the result of a 90° rotation.

DISCUSS

What is the relationship between the corresponding sides of the preimage and its image?

EXAMPLE B Triangle *BCD* was rotated to form its image, triangle $B'C'D'$. Identify the transformation and write a function to describe it.



1 Identify the coordinates of the vertices of both triangles.

The vertices of the preimage, $\triangle BCD$, are $B(-2, -5)$, $C(3, -1)$, and $D(6, -3)$.

The vertices of the image, $\triangle B'C'D'$, are $B'(-5, 2)$, $C'(-1, -3)$, and $D'(-3, -6)$.

2 Compare the triangles visually to identify the transformation.

From eyeballing the figures, it looks like $\triangle BCD$ was turned about $\frac{1}{4}$ clockwise around the origin to form $\triangle B'C'D'$. That is a $-90°$ rotation, which is the same as a 270° rotation.

3 Use the function for a 270° rotation to confirm your guess.

The function for a 270° rotation is $R_{270°}(x, y) = (y, -x)$. Apply this function to the vertices of $\triangle BCD$.

$R_{270°}(-2, -5) = (-5, 2)$ This is B'.

$R_{270°}(3, -1) = (-1, -3)$ This is C'.

$R_{270°}(6, -3) = (-3, -6)$ This is D'.

Each point (x, y) on $\triangle BCD$ has a corresponding point $(y, -x)$ on its image, so $\triangle B'C'D'$ is the result of a 270° rotation.

▶ The transformation is a 270° rotation, which can be represented by the function $R_{270°}(x, y) = (y, -x)$.

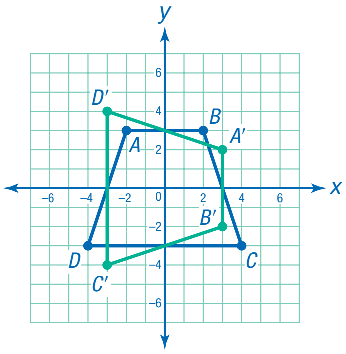
TRY

Identify and write a function to describe the rotation needed to move $\triangle B'C'D'$ back onto $\triangle BCD$. How does this notation compare to the notation for the 270° rotation?

Practice

Identify the number of degrees (45°, 90°, 180°, or 270°) by which each quadrilateral *ABCD* has been rotated about the origin to form its image.

1.



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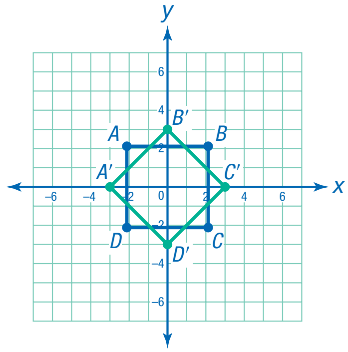
2.



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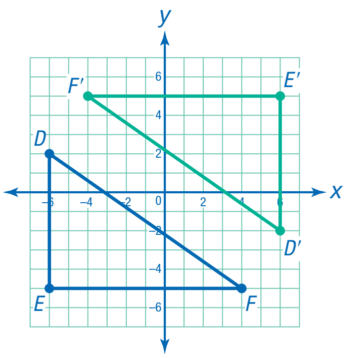
REMEMBER A −90° rotation is equal to a 270° rotation.

Describe how △*DEF* was rotated to form △*D′E′F′* both in words and in function notation.

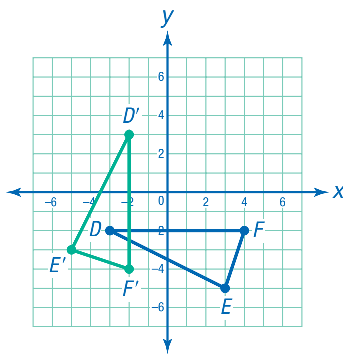
3.



Words: ________________________________

Function: ________________________________

4.



Words: ________________________________

Function: ________________________________

Write *true* or *false* for each statement. If false, rewrite the statement to make it true.

5. A circle is the set of all points that are equidistant from a point called the center.

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6. A quarter-turn in the counterclockwise direction is equivalent to a −90° rotation.

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7. Corresponding sides of a preimage and an image after a 270° rotation are parallel.

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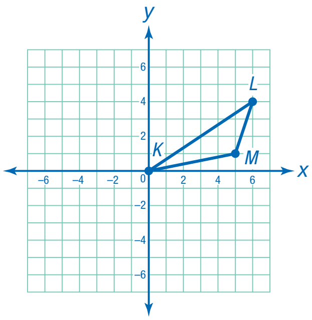
Use the given function to rotate $\triangle KLM$ to form $\triangle K'L'M'$. Identify the coordinates of the vertices of the image. Then identify the degree measure of the rotation.

8. $R_{\theta}(x, y) = (-x, -y)$



K'(___, ___) L'(___, ___) M'(___, ___)

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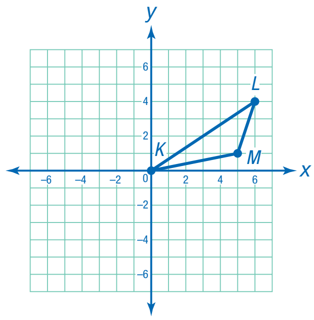
9. $R_{\theta}(x, y) = (-y, x)$



K'(___, ___) L'(___, ___) M'(___, ___)

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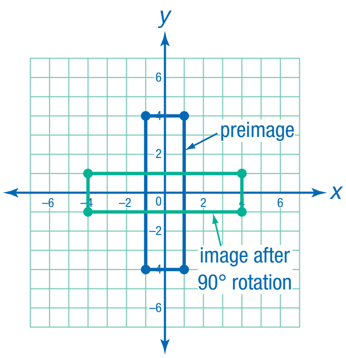
Solve.

10. **EXPLAIN** Sal drew a rectangle on a coordinate plane. He then rotated it 90° as shown below. Is there another way he could have rotated the rectangle that would have yielded the same image? Explain your reasoning.



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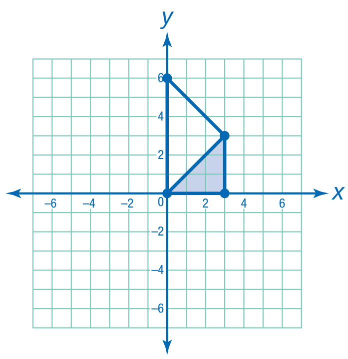
11. **DRAW** An artist drew a blue and white trapezoid on a computer. She wants to copy and rotate this image three times about the origin to create a figure that looks like a pinwheel. Describe three rotations she could use. Draw the pinwheel that would result from those three rotations.



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